

## TAKE THREE FROM FIVE (Answer)

Choose any five positive whole numbers.

Can you select three that will add up to a multiple of 3?

16  
20 3 7 1 1

For example:

If you start with 3, 7, 11, 16 and 20, you can select 3, 7 and 20 which add up to 30.

Choose another five positive whole numbers.

Can you select three again that will add up to a multiple of 3?

5  
1 1 3 18 10

Is it always possible to find three numbers that will add up to a multiple of 3 from any set of five positive whole numbers?

Can you explain why?

**Answer:**

Firstly, each one of the five integers can be expressed in the form  $3x+c$ , where  $x$  is an integer and  $c$  is either 0, 1 or 2. When we take three integers (say  $3x+c_1$ ,  $3y+c_2$ ,  $3z+c_3$ ) from a set of 5 and add them together, we will always get an integer in the following form:

$$3x+3y+3z+k$$

which equals

$$3(x+y+z)+k$$

(where  $k$  is the sum of  $c_1$ ,  $c_2$  and  $c_3$ )

It is obvious that the term  $3(x+y+z)$  must be a multiple of 3. Therefore, we only need to consider  $k$  in order to see whether  $3(x+y+z)+k$  is a multiple of 3.

For  $3(x+y+z)+k$  to be a multiple of 3,  $k$  must therefore also be a multiple of 3.

As  $c = 0, 1$  or  $2$  therefore there are 4 different ways in which  $k$  can be a multiple of 3:

1.  $0 + 0 + 0$
2.  $1 + 1 + 1$
3.  $2 + 2 + 2$
4.  $0 + 1 + 2$

Out of a group of 5 integers, it is always possible for three values of  $c$  to add up to a multiple of 3. This is because in a group of 5 zeros, ones or twos, there is either at least one of each number  $(0,1,2)$ , or at least 3 of just one of the numbers  $(0,0,0$  or  $1,1,1$  or  $2,2,2)$ .

Therefore it is always possible to choose three numbers that will add up to a multiple of 3 from any group of 5 numbers.